

Simulation of Current Experiments on the Tunneling Effect of Narrow ϵ -Near-Zero Channels

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In this paper, we discussed the current experiments on the tunneling effect of electromagnetic energy through narrow channels of ϵ -near-zero(ENZ) medium in the microwave range. Using the finite element method, we carried out a full wave simulation of the two kinds of experimental configurations at present. It was shown that the ability of the electromagnetic waves penetrating into the ENZ medium is very necessary. The present experimental setups only using metamaterial-filled narrow channel in the configuration of parallel plate waveguides are unlikely to realize effective tunneling. Contrarily, the artificial plasma medium emulated using hollow metallic waveguide can achieve nearly perfect tunneling in a narrow channel without any transition section around its cutoff frequency, which exceed the scope of original ENZ tunneling theory and can be described by a simple equivalent circuit model.

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I. INTRODUCTION

Since the rediscovery of the theory first proposed by Veselago in 1968[1], metamaterials have been an attracting subject of growing worldwide interest. The metamaterials can possess many exotic electromagnetic properties which are absent in natural materials. So they are of particular importance in the engineering of electromagnetic wave fields, which is leading to a variety of new microwave or optical devices.

Most current research is still focused on the double negative (DNG) metamaterials in which the permittivity and permeability can both less than zero simultaneously, leading to the negative index of refraction. Recently, there is a surge of investigation on ϵ -near-zero(ENZ) medium whose effective permittivity ϵ is near zero and effective permeability μ remains natural. In 2006, Silveirinha and Engheta[2] have theoretically predicted that the electromagnetic waves can be squeezed and tunnel through very narrow subwavelength channels filled with ENZ medium, independent of the specific geometry of the channel. The incoming wave front can be replicated at the output interface. The reflection at bends or junctions of different devices can decrease greatly and even disappear using this method. So, the ENZ medium will play an important role in the applications of transport and interconnects of microwave and optical wave energy.

Till now, several experiments have been reported to testify Silveirinha and Engheta's ENZ theory and the seemingly 'counterintuitive' tunneling effects are demonstrated at the certain microwave frequencies[4, 5, 6, 7]. The tunneling geometries in these experiments are basically 'U-shaped'. At present, there are two ways to experimentally realize the ideal ENZ tunneling channels proposed by Silveirinha and Engheta.

In the work by Liu and Cheng et al[4, 5], the relatively complex artificial resonant inclusions implanted in the parallel plate waveguides or microstrip circuits are adopted to achieve the effective ENZ medium near the

resonance frequency. The two-dimensional(2D) scattering of the electromagnetic wave in the original theory are simulated by the incident polarized nearly transverse electromagnetic(TEM) waves.

In the later experiments, the other way was adopted by Edwards et al [6, 7]. The artificial plasma medium emulated by the TE_{10} guided mode propagating in hollow rectangular metallic waveguides, mimics the equivalent response of ENZ materials near the cut-off frequency of guide wave modes.

In all above experiments the transition section in which the incident mode "penetrates" into the narrow channel is less considered. Intuitively, the sharp discontinuities will cause strong reflection of electromagnetic wave and excite high order evanescent modes, which not considered in the discussion in current work. To investigate this problem, we performed the full wave simulations of these experiments respectively in the following sections. From these results, we emphasis the importance role of the transition section in the tunneling effect of the 2D ENZ channel. The experiments results must be cautiously examined to exclude other possibilities before being affirmed to the ENZ tunneling phenomena predicted by the theory. While for the case of hollow metallic waveguides, The effective tunneling can be realized and the transition section is unnecessary. The corresponding pictures of underlying physics are discussed.

II. ENZ CHANNEL BETWEEN PARALLEL PLATE WAVEGUIDES

Over the past years, the development of metamaterials has greatly expanded the realm of available permittivity ϵ and permeability μ [8]. The artificially microstructured composites, may have become the most straight way to realize the effective medium with certain electromagnetic properties, including ENZ. In the work by Liu and Cheng [4, 5], the planar structure of complementary split ring resonator (CSRR) are adopted, which have an electric

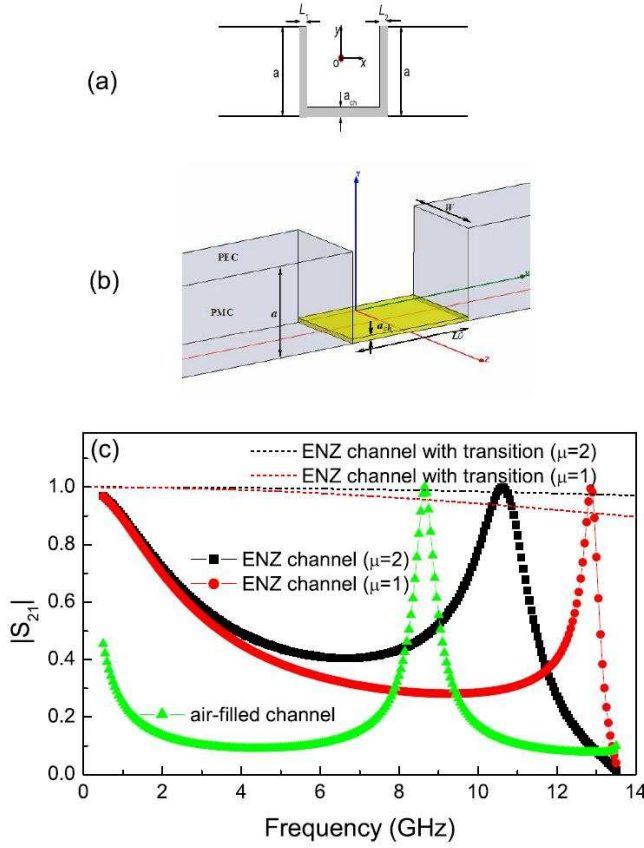


FIG. 1: (a) A U-shaped geometry of the microwave tunneling problem in a 2D parallel plate waveguide theoretically discussed by Silveirinha and Engheta in Ref[3]. (b) Setup of our simulation, PMC boundaries conditions are on the $x=0$ and w planes. Other outer faces except two waveports are assigned as PEC boundaries. $a = w = 11\text{mm}$, and $a_{ch}=1\text{mm}$. (c) Simulation results of (b): the transmission coefficients $|S_{21}|$ of ENZ channels with the same geometry ($L_0 = 15\text{mm}$) and different permeabilities ($\mu/\mu_0 = 1, 2$). The one for a air-filled channel is also plotted. As a comparison, $|S_{21}|$ for the ENZ($\mu/\mu_0 = 1, 2$) tunneling model of the same size with side ENZ transition section ($L_1 = L_2 = a_{ch}$) is plotted as dashed lines.

resonance in the certain range of frequency[9]. The resonance permittivity values of effective medium can be achieved.

A general U-type geometry for the 2D tunneling problem of a ENZ channel(shown in Figure 1(a)) has been discussed in detail as a prototype in detail in Silveirinha and Engheta's theory[3]. Although contributing to the total cross section ($A_p = (L_1 a_1 + L_2 a_2) + L a_{ch}$) and inconveniencing the approximate condition ($k_0 \mu_r A_p \rightarrow 0$ in Ref[2]) to realize effective tunneling, the lateral ENZ sides is retained as transition section, allowing the electromagnetic wave to penetrate into the narrow channel.

But in current experimental setup[4, 5], only channel filled with metamaterials are considered, without any transition section of effective ENZ medium. The lateral

metallic step sides are between parallel plate waveguides. Although in the paper by Liu et al, they are considered as a shunt admittance in a simple equivalent model[14]. But in their approximation, it was oversimplified and can only be valid in a very limited frequency range. The inhomogeneities at the metallic step in waveguides are much more complicated. Furthermore, the incident TEM-mode waves in parallel plate waveguide can excite more high order evanescent modes, at each sharp metallic step, leading to the interaction between these discontinuities, which can't be reckoned in such a simple model. As we know, whether the tunneling effect can be realized in this kind of configuration has not been discussed theoretically yet.

Now we first consider the simplified ideal model, as shown in Fig. 1(b). Here the two planes, $x = 0$ and w , are assigned as perfect magnetic conducting(PMC) boundaries. Other outer faces except two waveports are assigned as perfect electric conducting(PEC) boundaries. These constraint conditions insure the incident waves are of transverse electromagnetic(TEM) mode with certain polarization as marked in (b). The field invariance along the z direction can be fulfilled, so it degenerates to a 2D problem independent of the width, w . If the effective medium theory holds well for the volume with the CSRR or other composite structures, this will be a ideal approximation for the experiments by Liu and Cheng et al [4, 5].

In our full wave simulation, the fixed height of the ENZ channel ($a_{ch}=0.5\text{mm}$) is much less than the y spacing of the input and output parallel plate waveguides, $a = 11\text{mm}$. Simulations of this parallel waveguides geometry were performed with Ansoft's High Frequency Structure Simulator (HFSS) based on finite element method[18]. Two set of constitutive parameters ($\mu_r=1, 2$) for lossless ENZ materials and are used.

For a fixed channel with $L_0 = 15\text{mm}$, the simulated results of the transmission coefficients $|S_{21}|$ are shown in Fig. 1(c)(the lines with symbols). the air-filled channel without lateral are simulated, and corresponding $|S_{21}|$ also plotted for comparison.

The $|S_{21}|$ curves of the two ENZ channels without transition section are far below unity in the most range of microwave frequency, though they are above the level of air-filled channel. For comparison, we simulated the ENZ tunneling model shown in Fig. 1(a) with side transition ENZ sections ($L_1 = L_2 = a_{ch}$). The $|S_{21}|$ curves(dashed lines) suggest nearly perfect tunneling ($|S_{21}| \rightarrow 1$) through the the narrow channel with the same size, independent of the permeabilities.

In the Fig. 1(b), the tangential components of electric fields must be zero at the vertical surfaces of both side steps, which is strongly non-compatible with the propagating TEM mode in the two parallel plate waveguides. So the discontinuities obstacles the propagation of electromagnetic waves for air or ENZ-filled channel. On the contrary, the normal components of electric fields must be zero due to finite electric flux density inside the ENZ

surface, while the tangential ones are continuous across the interface. Thereby the lateral ENZ side layer let the normally incident TEM waves penetrating into the ENZ directly with no need of changing its configuration as shown in Fig. 1(a).

Another prominent character of the $|S_{21}|$ curves in the two ENZ channels without transition sections is the $|S_{21}|$ became very significant (near unity) around certain frequencies. The position of transmission peaks in ENZ channels depends strongly on the constitutive parameters, 12.8GHz i.e. and 10.6GHz for the ENZ medium with $\mu=1$ and 2 respectively. The transmission peaks also exist in air-filled channels at different frequencies. and These have studied in detail as the Fabry-Pérot oscillation[13]. Does the peaks of $|S_{21}|$ in the ENZ channels have the same origin? We further simulated six cases with the different channels lengths, $L_0 = 10$ to 60mm with the same permeability ($\mu=2$). And the results are shown in Figure 2(a).

According the theory of Silveirinha and Engheta, the anomalous tunneling effect is independent of geometry if only the ENZ channel is narrow enough. Here, the frequency of these transmission peaks increases as we decrease the length of channel section, which shows a strong dependence of geometry. (The resonance peaks also alter as we change the width of ENZ channel, a_{ch} (results are shown here). So the origin of the tunneling effect can be excluded.

To survey the underlying propagation processes in ENZ channels at the resonance frequencies, the distribution of the calculated electric field at 0 phase are depicted in Figure 2(b) for a typical channel with $L_0 = 15$ at the corresponding resonance frequency (10.6GHz). The magnitude of the calculated electric field along the central line of the channel (red line in Figure 1(b)) is also plotted in the Fig 2(c). The Fabry-Pérot-like character of the strong “standing” waves is observed in the ENZ channel with the maximum magnitude at the two open ends of the channel, very similar to Fabry-Pérot oscillation in the air-filled slots or channel[13].

Commonly, the m th-order Fabry-Pérot resonance occurs at the frequencies near $f = mc/(2nL_0)$ (here c is the light velocity in vacuum, h is the length) as the channel width is small enough. The transmission peaks will ‘blue shift’ as the length channel increases, which is consistent with our results. So these transmission peaks could be attributed to the Fabry-Pérot-like resonance.

There is a non-zero minimum of the magnitude of field in the middle of the channel, suggesting a slight difference from rigid standing wave picture of classical Fabry-Pérot resonance. It may seem more confusing because the refraction index ($n = \sqrt{\epsilon\mu}$) of a ENZ medium is near zero, and the wavelength of electromagnetic waves inside ENZ medium is ‘infinite’. So no obvious phase difference should exist in ENZ medium. The further thorough investigation on these anomaly resonance peak is being on the way.

These geometrical resonance characteristic of the

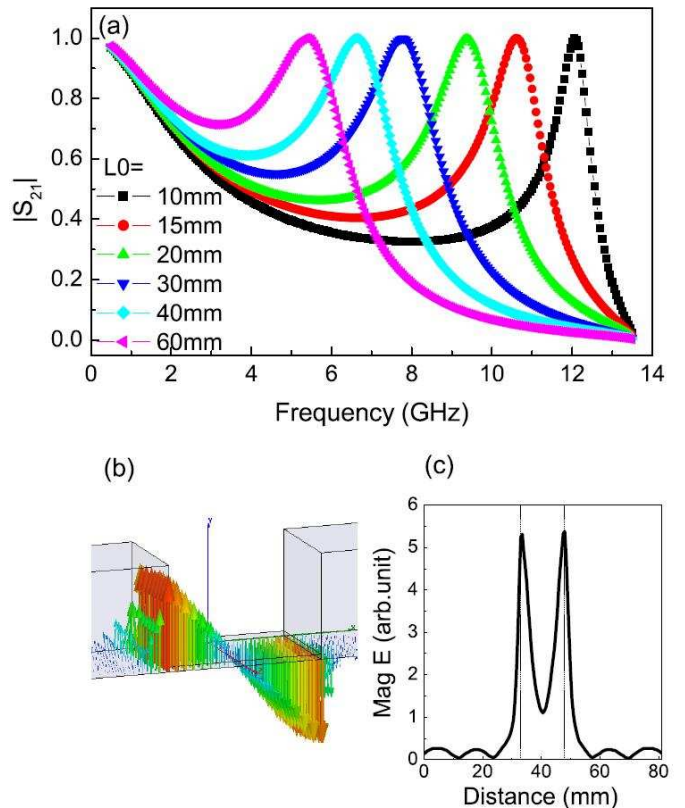


FIG. 2: (a) Simulated results of the transmission coefficients $|S_{21}|$ of ENZ tunneling channels with different length L_0 at a fixed phase (b) Calculated field distribution of electric vector for the $L_0 = 15$ mm ENZ channel at the corresponding resonance frequency, 10.6GHz. (snapshot in time). The corresponding magnitude of the electric field along the central line of the channel are shown in (c).

transmission peak is conceptually totally different from the tunneling effect associated with ENZ channels which is independent of specific geometry [2]. So in practice researchers can easily differentiate the resonance transmission peaks due to either the ENZ tunneling effect or these Fabry-Pérot-like one by altering the experimental geometry, such as the channel length L_0 .

In these present experiments [4, 5], the inhomogeneities also exists at the metallic vertical steps and the observed transmission $|S_{21}|$ at the passband is far below the perfect tunneling level. The geometries of the tunnel channel were fixed in their experiments. So the observed passband may not be identified as the results of ENZ tunneling. The geometries of the tunnel channel were fixed and never changed during their experiments. Thereby their results may not be the strong evidence of ENZ tunneling as they declared.

A full wave *ab initio* simulation is necessary to give a more clear picture. The configuration in our simulation (shown to the left of Figure 3(a).) are nearly the

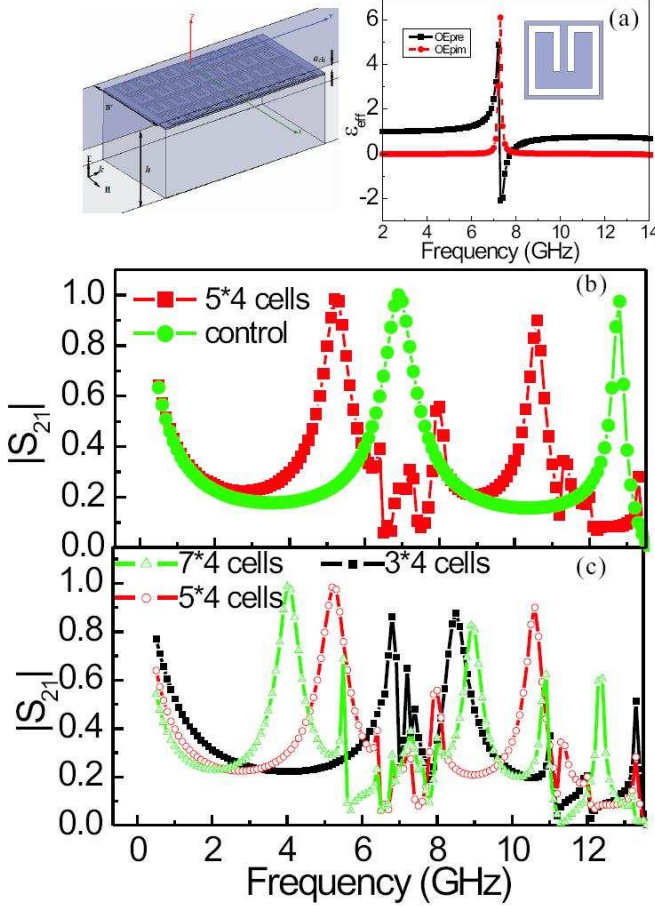


FIG. 3: (a) Retrieved effective permittivity for a CSRR unit cell (inset: the geometry of a CSRR cell). The simulation configuration for the tunneling experiment in Ref[4] are shown to the left of (a). The corresponding simulation results of transmission coefficients $|S_{21}|$ as a function of frequency are depicted in (b) and (c). Each tunnel channel consist of $m \times n$ uniform CSRR cells (m cells in the propagation direction and n cells in the transverse direction). The $|S_{21}|$ for control sample is also plotted for comparison.

same as the setup of the tunneling experiment in Ref[4], although with the less size in transverse direction than adopted in experiments (about 40 cells) due to finite our limited computing resource. The details of the CSRR cell structures (shown in the inset of Figure 3(a)) are the same as in Ref[4, 5], these CSRR patterns are perfect metallic faces on the surface of FR4 substrate (0.2 mm thick and $\epsilon/\epsilon_0 = 4.4$) in our setup. The retrieval result of effective permittivity for one unit cell was shown in the inset of Figure 3(a). The details of the retrieval procedure are described in Ref[4, 10].

In Figure 3(b), totally 5×4 CSRR cells are used (5 in the propagation direction and 4 in the transverse direction) to form the effective ENZ medium channel, and the simulated transmission $|S_{21}|$ vs frequency is depicted.

The un-patterned copper-clad FR4 board was used in the 'control' channel with the same geometry, which was also simulated and corresponding $|S_{21}|$ plotted for comparison. Apparently, the calculated $|S_{21}|$ curve for channel with CSRR patterns is similar to the control one. But the resonance peak shifts about 2GHz to lower frequency compared to the unpatterned control sample. There are weak fine resonance structures between 6 and 8 GHz, is consistent with the electric resonance found in single CSRR cell retrieval procedure (shown in Fig. 3(a)).

To find whether there exists strong tunneling evidence in the CSRR channel, we examined CSRR channels with different length (L_0). Three cases, 3×4 , 5×4 , 7×4 , are simulated, in which 3, 5 and 7 cells are along the longitudinal direction respectively. The corresponding results of transmission $|S_{21}|$ are depicted in Figure 3(c).

One can see that basically their behavior are similar and the peaks of the resonance transmission moves to low frequency as the channel length increases, which agrees with the previous Fabry-Pérot-like oscillation discussed in Figure 2(a). Apparently, the current geometry-sensible character of the resonance transmission maximums in these CSRR patterned channels didn't give any direct verification of the ENZ tunneling theory. In current experiments using the configuration of TEM parallel plate geometry, the transition section were neglected, and the observed transmission peaks were more like to be due to Fabry-Pérot-like oscillations. More work must be done to improve the current experiments to realize genuine verification of ENZ tunneling.

Additional, according to our results (shown in Fig. 3(c)), the fine structures due to the resonance CSRR inclusion also shifts as vary channel length. The departure from the retrieving results from one unit cell is very remarkable, suggesting the possible failure of the effective medium in CSRR inclusions for finite size and its inherent dispersion properties.

III. TUNNELING IN HOLLOW RECTANGULAR METALLIC WAVEGUIDE OPERATED AT THE CUT-OFF FREQUENCY)

A close analogy has been drawn between the propagating mode within a waveguide and the plane wave propagation in equivalent bulk medium with effective constitutive parameters[11]. Exploiting the inherent dispersion relation of hollow waveguides, people have realized plasma-like effective analogous to these of metamaterials.

For the fundamental transverse electric (TE_{10}) mode in rectangular metallic waveguide filled with non-magnetic medium, phase constant is as follows:

$$\beta = \sqrt{(n\omega/c)^2 - (\pi/w)^2} = \omega \sqrt{\mu_0 \epsilon_{eff}}$$

where ω is the angular frequency, μ_0 the magnetic permeability of vacuum and n the refraction index of medium filled in the waveguides. w is the fixed width of the

waveguides. So by analogy, the effective permittivity ϵ_{eff} can be written as,

$$\epsilon_{eff} = \epsilon_0(n^2 - c^2/(4f^2w^2))$$

c is the speed of light in air and f is the frequency. The cut-off frequency $f_0 = c\pi/w\sqrt{\epsilon}$ and the effective permeability remains μ_0 , independent of the operating frequency.

Thereby the TE_{10} mode can be used to emulate the propagation of a TEM wave in an classical lossless plasma medium. It has been used in artificial plasma simulation at microwave frequencies in Ref[11]. In 2002, by inserting metamaterials with effective negative permeability inside the rectangular waveguide below the cutoff(i.e. $\epsilon_{eff} < 0$), the equivalent double negative metamaterials have been realized.

In a series of recent work by Edwards et al[6, 7], many state-of-art demonstrations of nearly perfect tunneling have been realized, using the dispersive properties of rectangular metallic waveguides to mimic the response of the ENZ medium. For the U-type geometry similar to the experimental setup in Ref[6], our full wave simulation using HFSS gives nearly the same results(dashed lines shown in Fig. 4). Although the above simple equivalent analogy to ENZ medium lacks of convincing theoretical evidence. It is interesting that the configuration using rectangle waveguides can meet well with the ideal ENZ theory so well. The nearly perfect transmission around the cut-off frequency can be considered as a nice circumstantial evidence of the equivalence of ENZ medium and the waveguide.

Let's reconsider the origin derivation of their theory [2, 3] on the tunneling through narrow ENZ channel, the basic starting point is the z component of H (H_z) holds as constant in the ENZ channel. So in another word, if the $H_z = \text{const}$ condition can be fulfilled in the narrow channel in spite of specific filling, the tunneling effect can still occur without the precondition of ENZ medium.

In the TE_{10} mode in lossless rectangle metallic waveguides, the H_z of TE_{10} mode is written as[15],

$$H_z = jA_{10} \frac{\beta w}{\pi} \sin\left(\frac{\pi}{w}x\right) e^{-\beta z}$$

where A_{10} is constant coefficient and β is the propagation constant. Near the cut-off frequency, $\beta \rightarrow 0$, and $\nabla H_z \rightarrow 0$, So the $H_z = \text{const}$ condition are satisfied automatically at its cut-off frequency. So all the effects predicted by the tunneling theory [2, 3] can be realized. And the current present experiment can be thought as the more generic demonstration of the original tunneling theory of Silveirinha and Engheta.

From the discussion in Section II, without the transition section (L1 and L2), an ENZ channel in a parallel-plate geometry can't lead to effective tunneling and the Fabry-Pérot-like resonance peaks appear at certain frequencies instead. Is it the case for the hollow waveguide configuration? To investigate this problem, we perform the corresponding simulation with similar configuration without lateral transition sections as shown in Fig.

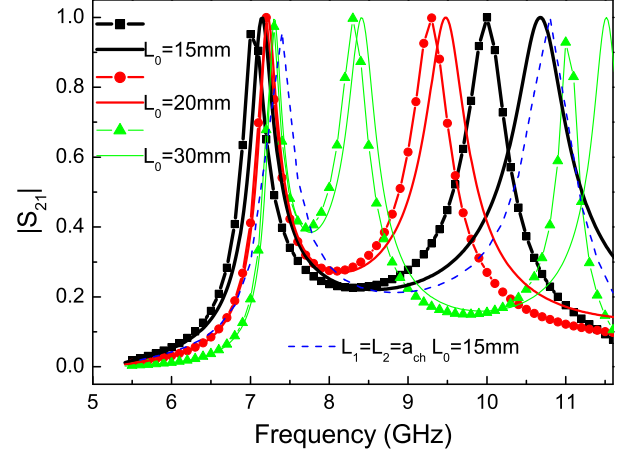


FIG. 4: Results of transmission coefficients $|S_{21}|$ of tunneling channels made of hollow metallic waveguides. The model is similar to Figure 1(b) with $a = 10\text{mm}$, $w = 20\text{mm}$, and $a_{ch} = 0.5\text{mm}$. All outer faces are PEC boundaries. Solid lines with symbols are simulated $|S_{21}|$ for different channel Lengths($L_0=15, 20, 30\text{mm}$) using HFSS. Similar solid lines without symbols are corresponding results calculated from a simplified equivalent circuit model(shown in Figure 6). The dashed line is simulated $|S_{21}|$ of the channel modelled with lateral transition sections($L_1 = L_2 = a_{ch}$).

1(b), in which two PEC plates parallel to the x - y plane at replaced the PMC ones as boundaries at $x = 0$ and w (we take $w = 20\text{ mm}$ here). The narrow channel are filled with air between two metallic rectangle waveguides filled with teflon($\epsilon \sim 2\epsilon_0$), supporting the propagation of TE_{10} mode above its corresponding cut-off frequency $f_0 = c/2w\sqrt{\epsilon} \approx 5.3\text{GHz}$.

Our simulation results are depicted in Fig. 4. Near the cut-off frequency of the air filled waveguide channel($f_0 = c/2w = 7.5\text{GHz}$), it is surprising that the $|S_{21}|$ curves for different channel length($L_0 = 15 - 50\text{ mm}$) shows the peaks of the resonance transmission around the cut-off frequency. The maximum of corresponding peaks approaches unity, i.e. the nearly perfect tunneling are achieved, independent of the geometry of the channel. No remarkable difference found with the comparison case with air-filled lateral transition section.

Although there are other transmission peak at higher frequencies, their resonance frequency are strongly geometrically dependent and Further analysis of corresponding field distribution(not shown in this paper) indicate they can attribute to Fabry-Pérot resonances.

To confirm the tunneling nature of transmission peak near cut-off frequency, the 3D field distribution of the electric field vectors in a typical channel($L_0 = 15\text{mm}$ and $a_{ch} = 0.5\text{mm}$) at its resonance frequency are plotted in Figure 5. The TE_{10} -mode-distributed field are much stronger than those in adjacent teflon filled waveguides,

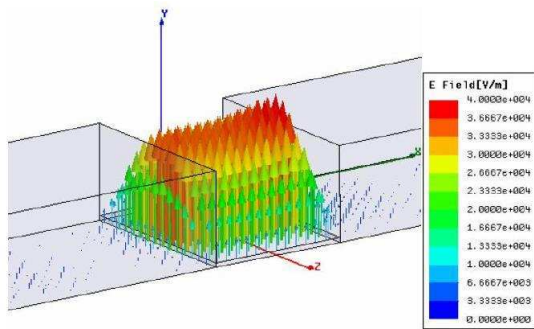


FIG. 5: Calculated field distribution of electric vector in the rectangle waveguide tunneling channel with length $L_0=15\text{mm}$ at the resonance frequency near its cutoff.

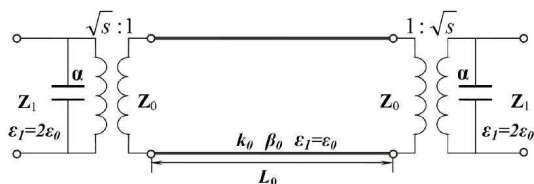


FIG. 6: Simplified circuit model for the metallic rectangle waveguide channel.

suggesting the huge squeezing of electromagnetic energy by this air-filled narrow channel. In the propagation direction, there is little change of the electric field which the channel. Thereby the phase difference between two end of the channel is near zero, as predicted in the ENZ tunneling theory[3].

Only the narrow air-filled channel itself between two waveguides can realize the effective tunneling of the incident TE_{10} electromagnetic mode. It seems very surprising because of the very different geometric and impedance mismatch between air-filled channel and two teflon-filled waveguide regions. So it is necessary to consider the E-plane steps discontinuities in metallic waveguides, which is an classical problem which can be simplified and analyzed using equivalent circuits model in Ref[16, 17]. To understand our current results, we modelled the whole system to be a simplified equivalent circuit (shown in Figure 6).

The fundamental TE_{10} mode is propagating in both

these waveguides with phase constants as follows,

$$\beta_i = \sqrt{\omega^2 \mu_0 \epsilon_i - (\pi/w)^2}$$

where the wave impedance is $Z_i = \omega \mu_0 / \beta_i$ respectively for each section ($i=0$ for air-filled channel and 1 for teflon-filled waveguides). And an ideal transformer[17] with the ratio $1 : \sqrt{s}$ ($s = a_{ch}/a$) was used in the model to reflect the change of impedance. The numerical calculation have been described [17] in detail. The calculated S_{21} for different L_0 have been shown in Fig. 4. The results agree very well with those from the full wave simulation using HFSS. The difference becomes slightly apparent only at high frequency.

Around the cut-off frequency, the phase velocity became infinite as $\beta \rightarrow 0$ and The change of E and H is along longitudinal direction. So L_0 is negligible. On the other hand, when $s \rightarrow 0$, the change of circuit impedance will be dominated by the two transformers (with the ratio $1 : \sqrt{s}$ and $1 : \sqrt{s}$) and relatively the effect of the shunt admittance can be neglected. (In our calculation, the moderate change on the value of the admittance load, α , have very weak effect on the position of resonance transmissions.) The super-coupling channel connected two waveguides with the same impedance can be obtained. The excellent agreements suggest that such a simple circuit model may grasp the nature of tunneling physics in metallic rectangular waveguides. Further experimental demonstrations of the tunneling in waveguides are expected in our future work.

IV. CONCLUSION

In summary, we systematically investigated the recent experiments about the demonstration of the tunneling effect through the narrow ENZ channel. It is shown that the ability of the electromagnetic waves penetrating into the ENZ medium is very necessary in the parallel-plate setup, and the transition section is indispensable to demonstrate effective tunneling effect. However for the experimental configuration made of hollow metallic waveguides, the nearly perfect tunneling can be achieved without transition section near its cutoff frequency, which is a more generic case and exceed the scope of the original ENZ tunneling theory.

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